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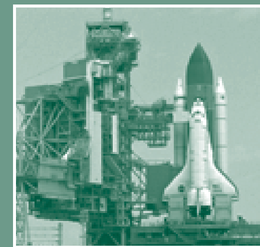

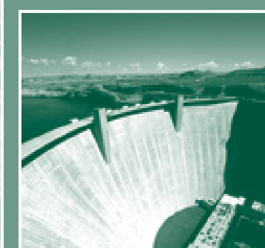

GAO COST ESTIMATING AND ASSESSMENT GUIDE

Best Practices for Developing and Managing Capital Program Costs

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LEARNING CURVE ANALYSIS

In this appendix, we describe the two ways to develop learning curves—unit formulation and cumulative average formulation—and discuss associated issues.

Unit Formulation

Unit formulation (or unit theory) states that as the quantity of units doubles, unit cost decreases by a constant percentage. It is represented by the formula

$Y = AX^b$, where

> Y = the cost of the Xth unit,
>
> A = the first unit (T1) cost,
>
> X = the unit number, and
>
> b = the slope coefficient (defined as the Ln (slope) / Ln (2)).

What causes the cost to decrease as the quantity doubles is the rate of learning, depicted by b in the equation. Stated more simply, if the slope were 80 percent, then the value of unit 2 would be 80 percent of the value of the 1st unit, the 4th unit would be 80 percent of the value of the 2nd unit, and so on. As the quantity doubles, the cost reduces by the learning curve slope.

Cumulative Average Formulation

Cumulative average formulation is commonly associated with T. P. Wright, who initiated an important discussion of this method in 1936.[14] The theory is that as the total quantity of units produced doubles, the cumulative average cost decreases by a constant percentage. This approach uses the same functional form as unit formulation, but it is interpreted differently:

$Y = AX^b$, where

> Y = the average cost of X units,
>
> A = the first unit (T1) cost,
>
> X = the cumulative number of units, and
>
> b = the slope coefficient (defined as above).

In cumulative average theory, if the average cost of the first 10 units were $100 and the slope were 90 percent, the average cost of the first 20 units would be $90, the average cost of the first 40 units would be $81, and so on.

The difference between unit formulation and cumulative average theory is in where the curve affects the overall cost. For the first few units, using cumulative average will yield higher cost savings than using

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[14] T. P. Wright, "Factors Affecting the Cost of Airplanes," *Journal of Aeronautical Science* 3:4 (1936): 122–28; reprinted in *International Library of Critical Writings in Economics* 128:3 (2001): 75–81.

a unit curve with the same slope. As the number of units increases, the difference between the results decreases.

Choosing between Unit Formulation and Cumulative Average

Choosing a formulation is not so much a science as an art. No firm rules would cause a cost estimator to select one approach over the other, but analyzing some factors can help decide which might best model the actual production environment. Some factors to consider when determining which approach to use are

1. analogous systems,
2. industry standards,
3. historic experience, and
4. expected production environment.

Analogous Systems

Systems that are similar in form, function, development, or production process may help justify choosing one method over the other. For example, if an agency is looking to buy a modified version of a commercial aircraft and unit curve were used to model the production cost for a previous version of a modified commercial jet, the estimator should choose unit formulation.

Industry Standards

Certain industries sometimes tend to prefer one method over the other. For example, some space systems have a better fit using cumulative average formulation. If an analyst were estimating one of these space systems, cumulative average formulation should be used, since it is an industry standard.

Historic Experience

Some contractors have a history of using one method over another because it models their production process better. The cost estimator should use the same method as the contractor, if the contractor's method is known.

Expected Production Environment

Certain production environments favor one method over another. For example, cumulative average formulation best models production environments in which the contractor is just starting production with prototype tooling, has an inadequate supplier base, expects early design changes, or is subject to short lead times. In such situations, there is a risk of concurrency between the development and production phases. Cumulative averaging helps smooth out the initial cost variations and provides overall a better fit to the data. In contrast, unit formulation is a better fit for production environments where the contractor is well prepared to begin production in terms of tooling, suppliers, lead times, and so on. As a result, there is less need for the data to be smoothed out by averaging the results.

There are no firm rules for choosing one method over the other. Choosing between unit formulation and cumulative average formulation should be based on the cost estimator's ability to determine which one best models the system's costs.

Production Rate Effects and Breaks in Production

Not only do costs decrease as more units are produced but also costs usually decrease as the production rate increases. This effect can be modeled by adding a rate variable to the unit learning formulation. The equation then becomes

$Y = AXbQr$, where

> Y, A, X, and b are as defined earlier,
>
> Q = production rate (quantity per time period or lot), and
>
> r = rate coefficient (Ln (slope) / Ln (2)).

This rate equation directly models cost reductions achieved by economies of scale. The rate at which items can be produced can also be affected by the continuity of production. Production breaks may occur because of program delays (budget or technical), time lapses between initial and follow-on orders, or labor disputes. Examining a production break can be divided into two questions:

- How much learning has been lost (or forgotten) because of the break in production?
- How will the learning loss affect the costs of future production items?

An analyst can answer the first question by using the Anderlohr method for estimating the loss of learning. The analyst can then determine the effect of the loss by using the retrograde method.

Anderlohr Method

When assessing the effect of a production break on costs, it is necessary first to quantify how much learning was achieved before the break and then to quantify how much of it was lost by the break. The Anderlohr method divides learning into five categories: personnel learning, supervisory learning, continuity of production, methods, and special tooling. Personnel learning loss occurs because of layoffs or removal of staff from the production line. Supervisory learning loss occurs when the number of supervisors is reduced because personnel have been reduced, so that supervisors who may no longer be familiar with the job are no longer able to provide optimal guidance.

Learning can also be lost when production continuity changes because the physical configuration of the production line has moved or optimization for new workers is lacking. Methods are usually affected least by production breaks, as long as they are documented. However, revisions to the methods may be required if the tooling has to change once the production line restarts. Finally, tools may break during the production halt or may not be replaced when they are worn, causing productivity loss.

Each category must have a weight assigned to capture its effect on learning. The weights can vary by production situation but must always total 100 percent. To find the percentage of lost learning—known as the learning lost factor—the estimator must determine the learning lost factor in each category and then calculate the weighted average (see table 71).

Table 71: The Anderlohr Method for the Learning Lost Factor

Category	Weight	Learning lost	Weighted loss
Personnel learning	30%	51%	0.1530
Supervisory learning	20	19	0.0380
Production continuity	20	50	0.1000
Tooling	15	5	0.0075
Methods	15	7	0.0105
Total learning lost	100%		0.3090 or 30.9%

Source: DOD.

In the table, if the production break were 6 months, the effect on learning would be almost a 31 percent reduction in efficiency, since the production line shut down.

Retrograde Method

Assume that 10 units had been produced before the production break. The true cost of the first unit produced after the production break would then equal the cost of the 11th unit—assuming no production break—plus the 30.9 percent penalty from the lost learning. The retrograde method simply goes back up the learning curve to the unit (X) where that cost occurred. The number of units back up the curve is then the number of retrograde or lost units of learning. Production restarts at unit X rather than at unit 11.

As illustrated by the Anderlohr and retrograde methods, costs increase as a result of production breaks. Cost estimators and auditors should question how the costs were estimated to account for learning that is lost, taking into account all factors that can be affected by learning.

Step-Down Functions

A step-down function is a method of estimating first unit production costs from prototype (or development) cost data. The first step is to account for the number of equivalent prototype units, based on both partial and complete units. This allows the cost estimator to capture the effects of units that are not entirely whole on the improvement curve. For example, if the development program includes a static article that represents 85 percent of a full aircraft, a fatigue article that represents 50 percent of a full aircraft, and three full aircraft, the development program would have 4.35 equivalent units. If the program is being credited with learning in development, the first production unit would then be unit 5.35.

After equivalent units have been calculated, the analyst must determine if the cost improvement achieved during development on these prototype units applies to the production phase. The following factors should be considered when analyzing the amount of credit to take in production for cost improvement incurred in development:

- the break between the last prototype unit and the start of production units,
- how similar the prototype units are to the production units,
- the production rate, and
- the extent to which the same facilities, processes, and people are being used in production as in development.

By addressing these factors, the analyst can determine proper placement on the curve for the first production unit. For example, analysis might indicate that cost improvement is continuous and, therefore, the first production unit is really the number of equivalent development units plus one. If it is further determined that the development slope should be the same as the production slope, the production estimate can be calculated by continuing down the curve for the desired quantity. This is referred to as the continuous approach.

Analysis of the four factors often leads the analyst to conclude that totally continuous improvement is not appropriate and that some adjustment is required. This could be because prototype manufacture was accomplished in a development laboratory rather than in a normal production environment or that engineering personnel were used rather than production personnel. Numerous reasons are possible for less than totally continuous cost improvement. Since all programs are unique, the analyst must thoroughly evaluate their particularities.

Two Theories Associated with Less Than Continuous Improvement

Two theories, sequential and disjoint, address the issue of less than continuous improvement. Both theories maintain that the improvement slope is the same in production and development but that a step down in value occurs between the cost of the first prototype unit and the cost of the first production unit.

In sequential theory, cost improvement continues where the first production unit equals the last production unit plus one, but a displacement on the curve appears at that point. In disjoint theory, the curve is displaced, but improvement starts over at unit one rather than at the last production unit plus one. These displacements are typically quantified as factors. Because disjoint theory restarts learning, it usually results in significantly lower production estimates.

The continuous cost improvement concept and sequential and disjoint displacement theories assume the same improvement slope in production as in development. Plots of actual cost data, however, sometimes indicate that production slopes are either steeper or flatter than development slopes. In cases in which the historic data strongly support a change in slope, the analyst should consider both a step down and a shift. For example, changing from an engineering environment to a heavily automated production line might both displace the improvement curve downward and flatten it.

End-of-Production Adjustments

As production ends, programs typically incur greater costs for both recurring and nonrecurring efforts. The recurring cost of end-of-production units is often higher than would have been projected from a program's historic cost improvement curve. This is referred to as toe-up. The main reasons for toe-up are

- the transfer of more experienced and productive employees to other programs, resulting in a loss of learning on the production line;
- reduced size of the final lot, resulting in rate adjustment penalties;
- a decrease in worker productivity from the psychological effect of the imminent shutdown of the production line;

- a shift of management attention to more important or financially viable programs, resulting in delayed identification and resolution of production problems;
- tooling inefficiency, resulting from tear-down of the tooling facility while the last production lot is still in process;
- production process modifications, resulting from management's attempts to accommodate such factors as reductions in personnel and production floor space; and
- similar problems with subcontractors.

No techniques for projecting recurring toe-up costs are generally accepted. In truth, such costs are often ignored. If, however, the analyst has access to relevant historic cost data, especially contractor-specific data, it is recommended that a factor be developed and applied.

Typically far more extensive than recurring toe-up costs are the nonrecurring close-out costs that account for the numerous nonrecurring activities at the end of a program. Examples of close-out costs are

- the completion of all design or "as built" drawings and files to match the actual "as built" system; often during a production run, change orders that modify a system need to be reflected in the final data package that is produced;
- the completion of all testing instructions to match "as built" production; and
- dismantling the production tooling or facility at the end of the production run and, sometimes, the storage of that production tooling.