CHAPTER 21

LEARNING CURVE

1. INTRODUCTION

It is a fundamental human characteristic that a person engaged in a repetitive task will improve his performance over time. If data are gathered on this phenomenon, a curve representing a decrease in effort per unit for repetitive operations can be developed. This phenomenon is real and has a specific application in cost analysis, cost estimating, or profitability studies related to the examination of future costs and confidence levels in an analysis. It could be used in estimating portions of a project, such as the production of magnets for the supercollider. This chapter discusses the development and application of the learning curve.

2. THE CURVE

The aircraft industry was the first to develop the learning curve. Based on comparison of manufacturing and aircraft industry learning curves, it is evident that a typical curve exists. It is an irregular line that starts high, decreases rapidly on initial units, and then begins to level out. The curve shows that there is progressive improvement in productivity but at a diminishing rate as the number produced increases. Figure 21-1 shows the appearance of the curve.

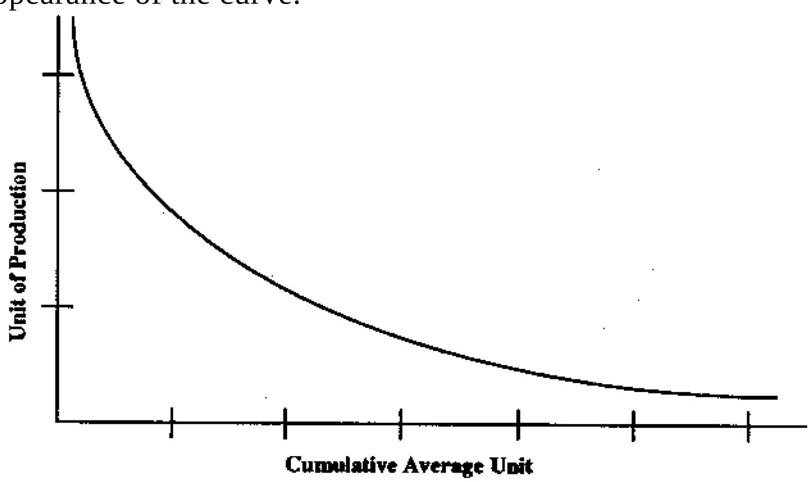

Figure 21-1. Curve Appearance

This suggests an exponential relationship between productivity and cumulative production. When this data is plotted on log-log paper, the data plots as a straight line. This suggests the relationship of the form:

$E_N = KN^S$

where E_N = effort per unit of production (i.e., manhours) to produce the Nth unit

K = constant, which is the effort to produce the first unit
s = slope constant, which is negative since the effort per unit decreases with production.

The above relationship will plot as a straight line on log-log paper.

Take the logarithms of both sides,

$$\log E_N = s \times \log N + \log K$$

which is the equation of a straight line

$$Y = sX + b$$

where $Y = \log E_N$, $X = \log N$, and $b = \log K$.

Figure 21-2 represents the data on log-log paper.

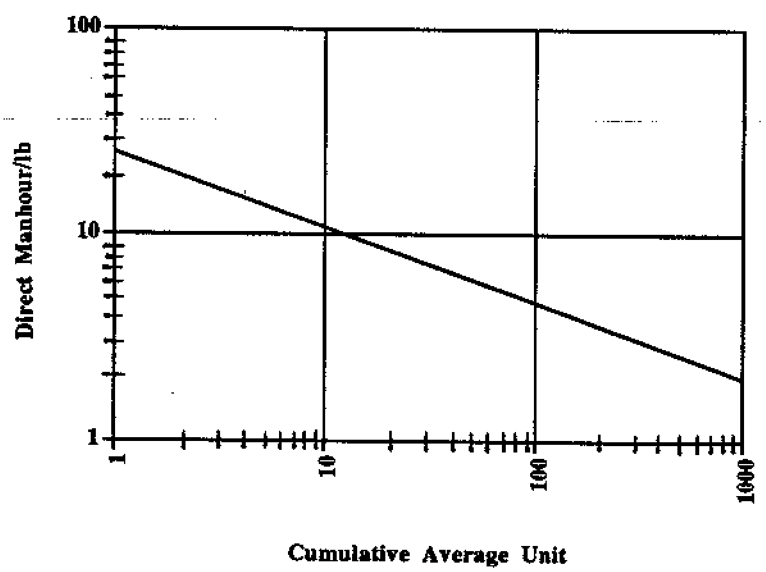

Figure 21-2. Data on Log-Log Paper

3. LEARNING CURVE FROM SINGLE-UNIT DATA

If the effort is available for each unit produced, any one of three curves can be plotted. They are the unit, the cumulative total, and the cumulative average. Following is Table 21-1, which includes single-unit data.

TABLE 21-1
PRODUCTION DATA

ITEM	UNIT HOURS	CUM. TOTAL HRS	CUM. AVG. HRS
1	10.0	10.0	10.0
2	8.0	18.0	9.0
3	7.3	25.3	8.4
4	6.3	31.6	7.9
5	6.0	37.6	7.5
6	5.6	43.2	7.2
7	5.6	48.8	7.0
8	5.0	53.8	6.7
9	5.1	58.9	6.5
10	4.5	63.4	6.3

From this data the unit, cumulative total, and cumulative average curves can be drawn.

A. Unit Curve

If a set of data is available for the effort required for single, individual units of production, the data can be plotted on log-log paper and the best line drawn with the eye. Having established the best line, any two points on the line can be used to determine, graphically or analytically, the slope of the line and K, which is the intercept at N = 1. This graphical method is quick, but it may require judgment when the data points are scattered.

The most accurate method for determining the best straight line is to use the least squares method.

B. Cumulative Total Curve

For this curve, the effort is described as cumulative total. This curve produces a line with a positive slope.

C. Cumulative Average Curve

The effort calculated for this curve is the cumulative average for each unit. It produces a curve that is usually a more regular curve than the unit curve.

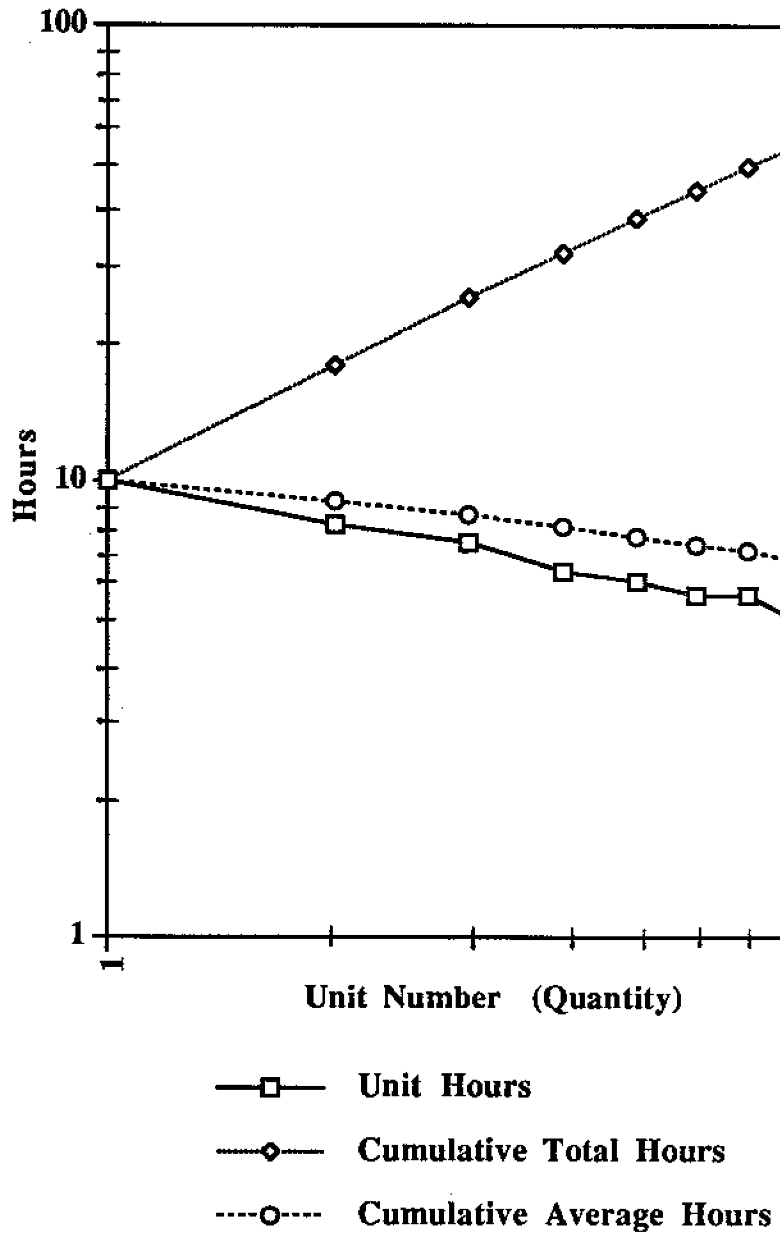

Figure 21-3. Curves on Log-Log Paper

4. EFFECTS OF DOUBLING PRODUCTION

The equation, $E_N = KN^s$, implies a constant fractional or percentage reduction in effort for doubled (or tripled, etc.) production. For example, for any fixed value for s gives:

$$E_1 = K(1^s)$$
$$E_2 = K(2^s)$$

$$\frac{E_2}{E_1} = \frac{K(2^s)}{K(1^s)} = 2^s$$

also,

$$E_2 = K(2^s)$$
$$E_4 = K(4^s)$$

$$\frac{E_4}{E_2} = \frac{K(4^s)}{K(2^s)} = 2^s$$

Every time production is doubled, the effort per unit required is a constant 2^s of what it was. It is common practice to express the learning-curve function in terms of the gain for double production. Thus, a 90 percent learning-curve function requires only 90 percent of the effort per unit every time production is doubled.

5. LEARNING CURVE TABLES

The learning curve will vary on different programs. A table of percentages for each type of program can be developed by taking the ratio of the average hours for the total program to the average hours for the first half of the total units. For example, the average hours for 200 units is 80 hours, and the average hours for the first 100 units was 100 hours each. Thus, 80/100 equals 80 percent. This is an 80 percent curve. Various curves are classified as 80 percent, 86 percent, 90 percent, etc., curves. If an 80 percent curve were plotted on arithmetic paper, we would expect a different-shaped curve for each project. If it is plotted on log-log paper, all curves will be straight lines. Tables for the various percentage curves can be developed so accurate figures can be calculated without using the more complicated mathematical formulas.

6. LEARNING CURVE FROM GROUPED DATA

Usually data are not available for the effort required to produce a single, individual unit. Instead, data are available for the average effort to produce a group or lot of units. From this the effort per unit for a lot can be calculated, but the effort per unit is not known. Before this data can be plotted, it is necessary to take each group and convert it to a point. This point within the group is associated with a “unit” number. The point can be referred to as the lot midpoint or the lot equivalent point. One method used to calculate the lot midpoint is to use the arithmetic mean of the first and last unit number in the lot. Once the lot midpoint is calculated, the curve can be drawn.

7. APPLICATION OF THE LEARNING CURVE

When estimating a project cost when one of the variables is a large quantity of a unit, the learning curve can be used. If the cost of production of the first units is known or if a percentage table can be assumed, the impact of several units being produced can be calculated. The effort can be defined as the cost to produce the item, and the cost impact can be evaluated.